# Resistance and Resistivity

**Objectives**

After going through this lesson, the students will be able to:

- Distinguish between resistance and resistivity and current and current density
- Define conductivity
- Derive a relation between current density, conductivity and electric field
- Effect of temperature on resistance and resistivity
- Know about carbon resistors and their colour coding

**Content Outline**

- Syllabus
- Module Wise Distribution
- Words You Must Know
- Introduction
- Difference between Resistance and Resistivity
- Relation between Current Density, Conductivity and Electric Field
- Factors affecting Resistivity
- Effect of Temperature on Resistance and Resistivity
- Electrical Energy and Power
- Solved Problems
- Problems for Practice
- Summary

## Unit Syllabus

Electric current, flow of electric charges in a metallic conductor, drift velocity and mobility, and their relation with electric current; Ohm's law, electrical resistance, V-I characteristics (linear and nonlinear), electrical energy and power, electrical resistivity and conductivity, Carbon resistors, colour code of carbon resistors; series and parallel combinations of resistors; temperature dependence of resistance. Internal resistance of a cell, potential difference and emf of a cell, combination of cells in series and in parallel.

Kirchhoff's laws and simple applications, Wheatstone bridge, Metre Bridge, Potentiometer-principle and its application to measure potential difference and for comparing emf of two cells; measurement of internal resistance of a cell.

## Module Wise Distribution - 08 Modules

The above unit has been divided into 8 modules for better understanding.

| | |
|---|---|
| Module 1 | • Electric current,<br>• Solids liquids and gases<br>• Need for charge carriers speed of charge carriers in a metallic conductor<br>• Flow of electric charges in a metallic conductor<br>• Drift velocity,<br>• Mobility and their relation with electric current |
| Module 2 | • Ohm's law,<br>• Electrical resistance,<br>• V-I characteristics (linear and nonlinear),<br>• Electrical energy and power,<br>• Electrical resistivity and conductivity<br>• Temperature dependence of resistance |
| Module 3 | • Carbon resistors,<br>• Colour code for carbon resistors;<br>• Metallic Wire resistances<br>• Series and parallel combinations of resistors<br>• Grouping of resistances<br>• Current and potential differences in series and parallel circuits |
| Module 4 | • Internal resistance of a cell,<br>• Potential difference and emf of a cell,<br>• Combination of cells in series and in parallel.<br>• Need for combination of cells |
| Module 5 | • Kirchhoff's Rules<br>• Simple applications. of Kirchhoff's Rules for calculating currents and voltages<br>• Numerical |

| | |
|---|---|
| Module 6 | ● Wheatstone bridge<br>● Balanced Wheatstone bridge condition derivation using Kirchhoff's Rules<br>● Wheatstone bridge and Metre Bridge.<br>● Application of meter bridge |
| Module 7 | ● Potentiometer –<br>● Principle<br>● Applications to<br>● Measure potential difference<br>● Comparing emf of two cells;<br>● Measurement of internal resistance of a cell.<br>● Numerical |
| Module 8 | ● Numerical<br>● Electrical energy and power |

## Words You Must Know

- **Resistance (R)**: In simple words, a 'resistance' can be anything which opposes/ obstructs the flow of current. The free electrons (in a conductor) constantly collide with its atoms / ions. These collisions are the basic cause of its 'resistance' to 'current flow'. It is measured as potential difference across a conductor per unit current flowing through it.

$$\frac{V}{I} = R$$

- **Oscillations**: A periodic to and fro motion about a fixed point.

- **Amplitude**: It is the maximum displacement, from the mean point.
- **Drift Velocity ($V_d$):** It is the average velocity acquired by a free electron, in a conductor under the influence of an external electric field. The magnitude of this velocity is very small, of the order of $10^{-4}$ m/s hence the name 'Drift Velocity'.

- **Relaxation time (τ):** It is the average time between two successive collisions of the free electrons. During this time the electron gains energy which is lost in the collision, from the applied external electric field.

- **Number density (n):** It equals the number of free electrons per unit volume.

- **Mobility (μ)**: For a charge carrier, responsible for current flow, it is defined as the magnitude of drift velocity of the charge carrier per unit applied electric field.

- **Ohm's law**: It states that the current flowing through a conductor is directly proportional to potential difference across the two ends of the given conductor, provided its temperature and other physical conditions remain constant.

$$\frac{V}{I} = R$$

  Here R is the resistance of the conductor, which is constant for a given conductor, under given conditions.

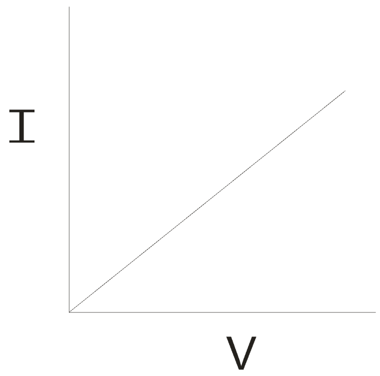
## Introduction

Ohm's law relates the potential difference, current and resistance. It states that the current flowing through a conductor is directly proportional to potential difference across the two ends of the given conductor, provided its temperature and other physical conditions remain constant.

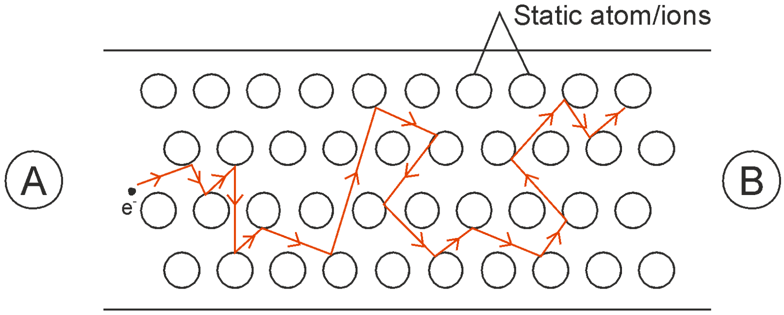
$$I \propto V$$

Or $$\frac{V}{I} = R$$

Where, R is the resistance of the conductor, which is constant for a given conductor, under given conditions.



## Difference in Resistance And Resistivity




**The figure illustrates the motion of a free electron within the conductor**

The basic knowledge of chemistry tells us that each substance is made up of atoms. The metals have a large number of free electrons which are in constant motion and collide with one another as well as with atoms or ions which fall in their way. They rebound, again collide, and again rebound.

These collisions of electrons with one another, with the atoms and /or ions, are the cause of resistance.

Resistance of a conductor is

i) directly proportional to the length of conductor $R \propto L$

ii) inversely proportional to its area of cross-section $R \propto 1/A$

Resistance $R = \rho L/A$

Where, ρ = resistivity or specific resistance

**Factors affecting Value of Resistance R:**

**(1) Dimensions of Conductor**

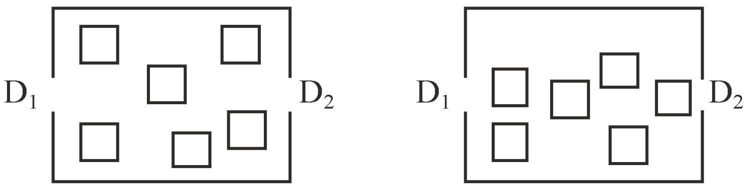
**(2) Nature of Material of Conductor**

Besides depending on the dimensions and nature of the conductor, resistance of the given conductor also depends on the temperature of the conductor.

## Resistivity/ Specific Resistance

It equals the resistance of a cube of the given material of side 1m. Resistivity is defined as **the resistance of a conductor of length 1m and cross-sectional area 1m$^2$**.

Consider two rooms having the same amount of furniture, as shown. The arrangement and orientation of furniture in the two rooms are, however, different




(i) (ii)

If a person wants to go from door 1 ($D_1$) to door 2 ($D_2$) Room (i) offers a lower 'resistance' to the movement than room (ii). This is because of the availability of a 'greater free space' along the path of the movement.

Thus resistivity solely depends upon the nature of material. So, it is constant for a given material.

From equation **R= ρ L /A**

we can write

$$\rho = R\frac{A}{L}$$

**The SI units of resistivity are ohm-meter or Ω-m.**

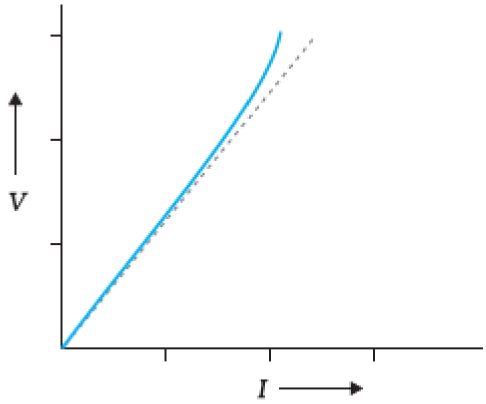
**The basic difference between resistance and resistivity is that resistance depends upon the nature of the material as well as the dimensions of conductor whereas resistivity depends only upon the nature of material.**

**For example:** Two copper wires have the same length, one wire is thicker than the other. The resistance of the thicker wire is less than the thin wire, but the resistivity of both wires is same as the material of two is same.

**Non-ohmic 'Conductors' (Limitations of Ohm's Law)**

There are some 'conductors' which do not obey Ohm's law; (V/I) is not constant for such materials. However the relation, V=IR, can still be used for both ohmic and non-ohmic conductors. The V-I characteristics curves, of different conductors, are of one or more of the following types:-

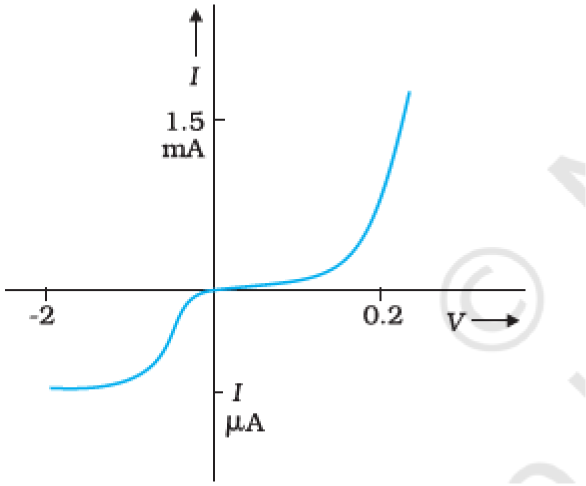
(1) V and I graphs are non-linear also.




The dashed line represents the linear Ohm's law. The solid line is the voltage *V* versus current *I* for a good conductor.

The deviation of the graph from a straight line nature is due to the fact that when current flows, the temperature of the conductor rises. For moderate temperature ρ changes linearly but at higher temperatures, ρ increases exponentially.

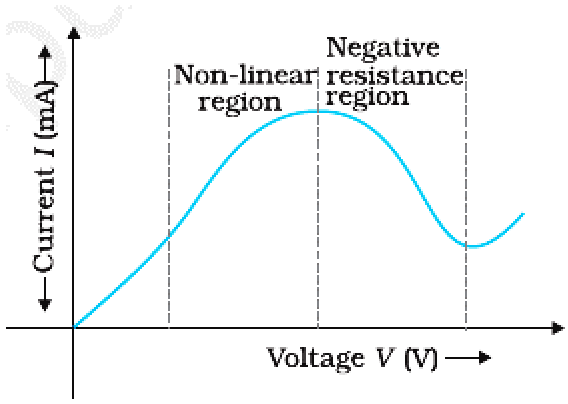
(2) The relation between V and I depends on the sign of V.




Characteristic curve of a diode. Note the different scales for negative and positive values of the voltage and current.

For +0.2V, the current is say $I_1$, but for -2V current is $I_2$ which is much smaller than $I_1$. So, the relation between V and I, varies with the sign of V.

(3) The relation between V and I is not unique, because there is more than one value of V, for the same value of current I.




Variation of current versus voltage for GaAs.

The value of current I remains the same for two different values of potential, $V_1$ and $V_2$. The value of V, for a given value of I, is not unique.

**One can note that the (V-I) graphs of ohmic conductors start from the origin and are straight lines, inclined at a certain angle to the axes. The graphs of non-ohmic conductors may or may not start from the origin and are non-linear.**

## Relation Between Current Density, Conductivity and Electric Field

**Current Density (J)**

The current density, J in the conductor is defined as the current per unit area of the surface held perpendicular to the flow of current.

$$J=\frac{I}{A}$$

(I = current, A = area of cross section, normal to the flow of current)

Current density has SI units of **$A/m^2$**. The current density is a vector quantity with direction along the direction of the current/ applied electric field.

When the normal to the cross sectional surface, makes an angle θ with the direction of the flow of current, we would have

$$J = I/A\cos\theta; \text{ when area making angle } \theta \text{ with current}$$

$$I = JA\cos\theta$$

$$I = \vec{J}.\vec{A}$$

Here $\vec{A}$ is area vector and its direction is normal to the surface while $\vec{J}$ **is** the current density vector.

**Conductance (G)**

It is the reciprocal of Resistance.

$$G = \frac{1}{R}$$

Its S.I unit is **Siemen**, or **mho**, or **ohm$^{-1}$**

**Conductivity (σ)**

It is the reciprocal of resistivity.

$$\sigma = 1/\rho$$

Its S.I unit is ohm$^{-1}$m$^{-1}$ or siemen$^{-1}$ m$^{-1}$

**Relation between J, σ and E**

We have

$$V = IR = I\left(\frac{L}{A}\right)$$

Therefore $$V/L = I\rho/A$$

$$E/\rho = I/A$$

$$\vec{J} = \boldsymbol{\sigma}\,\vec{E}$$

**This is vector form ohm's law**

## Factors Affecting Resistivity

As we know that that the resistance R of a wire of a given material, is given as

$$R = \rho\frac{L}{A}$$

(L = the length of the wire; A= area of cross section of the wire, ρ = resistivity)

We also know that

$$\text{Resistance } R = \frac{mL}{ne^2\tau A}$$

Hence

$$R = \rho\frac{L}{A} = \frac{mL}{ne^2\tau A}$$

Therefore, Resistivity $$\rho = \frac{m}{ne^2\tau}$$

**Resistivity of a material depends on**

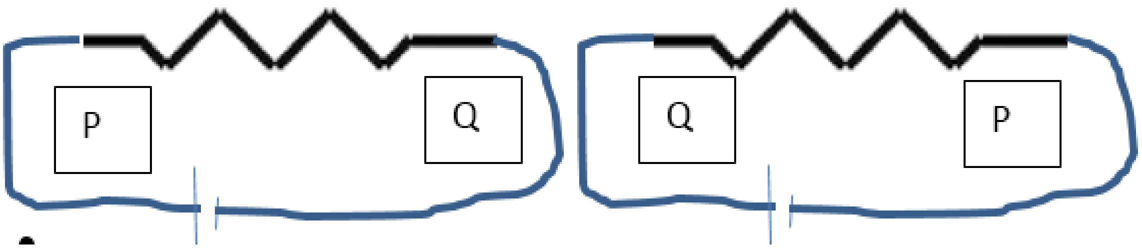
- $\rho \propto m$ **; mass of the charge carrier m**

- $\rho \propto 1/n$ ; Number density of charge carriers (n) depends on the nature of the material ;
- $\rho \propto 1/e^2$; Charge on the charge carrier e
- $\rho \propto 1/\tau$ ; Average relaxation time (τ) is a function of temperature. As the temperature increases, τ decreases and resistivity increases; hence resistivity depends on temperature.
- Resistivity increases linearly with temperature over a moderate temperature range and it varies exponentially for higher temperature ranges.

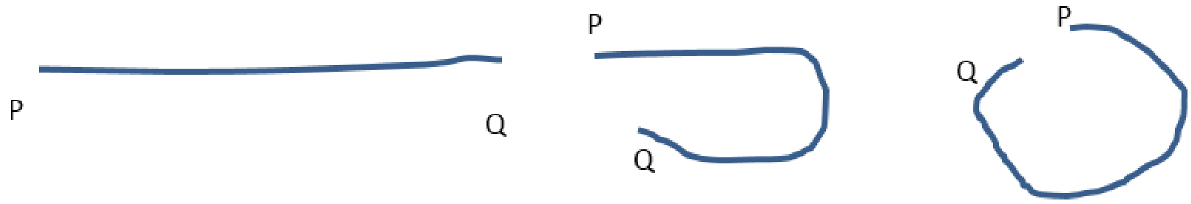
## Misconceptions About Resistances

- Resistances are conductors and not insulators.
- An insulator has indefinite resistance.
- Resistance wires are straight and not zig zag , in circuit diagrams we represent them by a zig zag line

- Resistances do not have any fix  positive or negative terminal.
- They are neutral, however they may develop a potential gradient across them when connected to a battery.
- You can connect the resistance in any way you like, which means suppose a resistance R has ends P and Q




- Wrapping a length of wire does not change its resistance.




- The resistance changes if the two ends of a long wire are joined and placed in a


circuit

It will be as though two wires of half of original length are connected in parallel
If we were to calculate its value, we will need rules of addition of resistances in parallel

- Resistance of a wire of length (l) of homogeneous material (ρ) of uniform area (A) of cross section is given by

$$R = \rho\frac{l}{A}$$

- Different lengths of the same material and the same area of cross section wire will have different resistance.

- Wires of the same material have the same length but different areas of cross section have different resistance. specific wire gauge ( swg ) is a specification for a wire based on its diameter
- Wires of the same length have the same area of cross section but different material will have different resistances.

- If the resistance wire of a certain length, is of uniform density but non uniform area of cross section
  - Its specific resistance remains the same
  - Its resistance remains the same
  - Its resistance per cm will change

- **If a wire is stretched to increase its length**
    - **Its specific resistance remains the same as the material remains the same**
    - **Its resistance will change**
    - **Its volume will remain the same**
    - **Its diameter /area of cross section will decrease**

## Effect Of Temperature On Resistance And Resistivity

On increasing temperature, the resistivity of a conductor increases. This is because with increasing temperature the amplitude of oscillations of the atoms and ions (about their mean position) increases, hence the frequency of collisions of the free electrons increases. This results in decrease in relaxation time (τ), therefore resistivity ρ increases ($\rho \propto 1/\tau$).

The change in resistivity with change in temperature is given by:

$$\rho_2 = \rho_1(1 + \alpha\Delta T)$$

As we know resistance is directly proportional to the resistivity of a material, therefore if the temperature of conductor changes by ΔT, then change in resistance will be given as

$$R_2=R_1 (1+\alpha \Delta T)$$

The above relation is used only for moderate temperature rise

Here

α → temperature / Thermal coefficient of resistance

ΔT → Change in temperature

$\rho_2$, $R_2$ → final resistivity; resistance

$\rho_1$, $R_1$ →initial resistivity; resistance

Thermal or temperature coefficient of temperature (α) gives the fractional change in resistance per unit change in temperature.

$$\alpha = \frac{R_2-R_1}{R_1\times\Delta T}$$

The SI unit of α is $^oC^{-1}$

The value of α is different for different types of materials, like

(1) For metals α is positive; hence on increasing temperature the final resistance ($R_2$) increases.

$$R_2>R_1 \text{ (for metals)}$$

(2) For non-ohmic conductors like insulators, semiconductor and electrolytes $\alpha$ is negative, hence on increasing temperature, final resistance decreases.

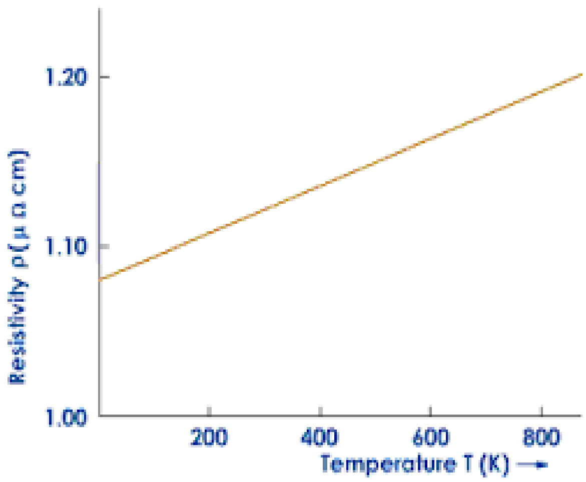
Hence, $R_2 < R_1$

(3) Metal alloys like manganin, constantan and nichrome are used for making resistances of high precision because:-

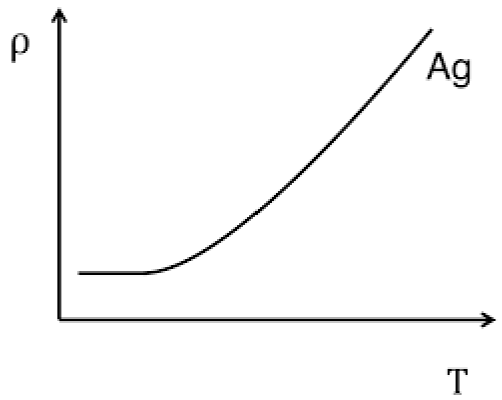
- They have high resistivity and
- 'α' is very small for them. The resistance does not change much with even several degree increases in temperature.

Resistivity increases linearly for moderate temperature rise, it varies exponentially for large temperature rises. The variation of resistivity for alloys, metals and semiconductors are shown below.

- **For alloys**

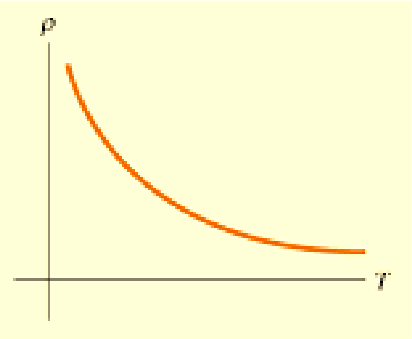



- **For metals**



Note that resistivity changes linearly only for moderate temperature range. For very low and very high temperatures it varies exponentially.

- **For semiconductors**




**Resistivities Of Some Materials**

| Material | Resistivity (Ω m) at 0°C | Temperature coefficient of resistivity, a $(°C)^{-1}$ |
|---|---|---|
| **Conductors** | | |
| Silver | $1.6 \times 10^{-8}$ | 0.0041 |
| Copper | $1.7 \times 10^{-8}$ | 0.0068 |
| Aluminium | $2.7 \times 10^{-8}$ | 0.0043 |
| Tungsten | $5.6 \times 10^{-8}$ | 0.0045 |
| Iron | $10 \times 10^{-8}$ | 0.0065 |
| Platinum | $11 \times 10^{-8}$ | 0.0039 |
| Nichrome ~ (alloy of Ni, Fe, Cr) | $100 \times 10^{-8}$ | 0.0004 |
| Manganin (alloy) | $48 \times 10^{-8}$ | $0.002 \times 10^{-3}$ |
| **Semiconductors** | | |
| Carbon (graphite) | $3.5 \times 10^{-5}$ | – 0.0005 |
| Germanium | 0.46 | – 0.05 |
| Silicon | 2300 | – 0.07 |

| Insulators | | |
|---|---|---|
| Pure Water | $2.5 \times 10^5$ | |
| Glass | $10^{10} - 10^{14}$ | |
| Hard Rubber | $10^{13} - 10^{16}$ | |
| Fused Quartz | $\sim 10^{16}$ | |

## Electrical Energy And Power

You must have observed that when current flows through a wire, for some time, the wire gets heated. Ever thought why is it so?

When current flows through a wire, the electrons collide with the atoms/ions and transfer their energy to atoms. Due to these collisions, a lot of energy is dissipated in the form of heat; this is why the wire gets heated up. The rate of dissipation of this energy is called power.

Let us now derive an expression for power

Let a charge q be taken from point A to point B in a given conductor in a time t.

Work done is $W = q\,\Delta V$

$W = q\,(V_B - V_A)$

Also, Power, $P = W/t$

Therefore $P = q\,\Delta V/t$

Or $P = VI$ $(as\ I = q/t)$

Using Ohm's law

$$V = IR,$$

we get

$$P = I^2R = V^2/R$$

This is the power loss ("ohmic loss") in a conductor of resistance R carrying a current I.

Where does this power come from? As we have reasoned before, we need an external source to keep a steady current flowing through a conductor. It is this source which supplies this power. In the case of a cell, it is the chemical energy of the cell which supplies this power for maintaining the flow of current.

Consider a device R, to which a power P is to be delivered, via transmission cables, having a resistance $R_c$. If V is the voltage across R and I the current through it, then,

$$P = VI$$

The connecting wires, from the power station to the device, have a finite resistance $R_C$. The power dissipated in the connecting wires, (which is wasted) is $P_c$ where

$$P_c = I^2 R_c$$

$$P_c = P^2 R_c / V^2 \qquad (as\ I = P/V)$$

$$P_c \propto 1/V^2 V_2$$

**Note, here P is the power delivered and $P_c$ is the power wasted.**

To save current carrying wires from excessive heat and to minimise the power loss, long distance transmission of electrical power is done at "**high voltage, low current**".

## Solved Examples

**Example**

A Potential difference of 3V is applied across a conductor of resistance 1.5Ω. Calculate the number of electrons flowing through it in one second. Given charge on an electron, e= $1.6\times10^{-19}$ C.

**Solution**

V = 3 V; R = 1.5Ω ; e = $1.6\times10^{-19}$C

I = V/R = 3/1.5 A = 2A

Also I = Q/t = ne/t

$n = It/e = 2\times1/ 1.6\times10^{-19} = 1.25\times10^{19}$

**Example**

Calculate the resistivity of the material of wire 1.0m long, 0.4mm in diameter and having a resistance of 2.0 Ω.

**Solution**

Here; L=1m

D= 0.4mm = $4\times10^{-4}$m

R =2Ω

Area of cross-section, A= $\pi (D^2 / 4)$

$A = \pi \times (4\times10^{-4})^2 / 4\ m^2$

$A = 4\pi \times 10^{-8}\ m^2$

$\rho = R A/ L = 2.0\times 4\pi\times10^{-8} / 1.0 \Omega\text{-m}$

$\rho = 2.514 \times 10^{-7}$ Ω-m

**Example:**

The heating element of an electric toaster is made of nichrome. When a very small current passes through it at room temperature 27 $^{O}$C, its resistance is 75.3Ω. When the toaster is connected to a 230V supply, the current settles after a few seconds to a steady value of 2.68A. Find the steady temperature of the nichrome element? (The temperature coefficient of resistance of nichrome over averaged over the temperature range is $1.7\times10^{-4}$ $^{O}C^{-1}$).

**Solution:** Given;

$R_{27}$ =75.3 Ω,

$\alpha = 1.7\times10^{-4}$ $^{O}C^{-1}$

V = 230V,

I = 2.68A

$R_t = (230 / 2.68)\ \Omega = 85.82\ \Omega$

Now, $t_2 - t_1 = (R_2 - R_1) / R_1 \times \alpha$

Therefore, $(t - 27) = (R_t - R_{27}) / R_{27} \times \alpha$

$$(t - 27) = (85.82 - 75.3) / 75.3 \times 1.7\times10^{-4}$$

$$(t - 27) = 822\ ℃$$

$$t = 822 + 27 = 849\ ℃$$

**Example**

The resistance of the platinum wire of a platinum resistance thermometer, at the ice point is 5Ω and at steam point is 5.23Ω. When this thermometer is inserted in a hot bath, the resistance of the platinum wire is 5.795Ω. Calculate the temperature of the bath.

**Solution:**

Given,

$R_0$ = 5Ω,

$R_{100}$ = 5.23Ω,

$R_t$ = 5.795Ω

t=?

We have $\alpha = (R_{100} - R_0) / ( R_0 \times 100)$ (1)

Also $R_t = R_o ( 1 + \alpha t)$

Therefore $t = \frac{R_t - R_o}{\alpha R_o}$ (2)

Using (1) and (2)

$$t = \frac{R_t - R_o}{R_o} \times \frac{R_o \times 100}{R_{100} - R_o}$$

$$t = 100 \times \left[\frac{5.795-5}{5.23-5}\right]^O C$$

$$t = \left[\frac{100 \times 0.795}{0.23}\right]^O C$$

$$t = 345.7^O C$$

## Questions for Practice

1. **Show on a graph, the variation of resistivity with temperature for a typical semiconductor.**
2. **Two wires of equal length, one of copper and other of manganin have the same resistance. Which wire is thicker?**
3. **Define resistivity of a conductor. Write its S.I. unit.**
4. **A cylindrical metallic wire is stretched to increase its length by 5%. Calculate the percentage change in its resistance?**
5. **Define the term 'temperature coefficient of resistivity'. Show on a graph the likely variation of resistivity with temperature for nichrome.**
6. **Draw a graph showing variation of resistivity of carbon with temperature.**

## Summary

- **Resistivity**(ρ): It equals the resistance of a cube of given material having a side 1m.It is defined as the resistance of a conductor of the given material, having a length 1m and a cross-sectional area of $1m^2$.

  Electrical resistivity of substances varies over a very wide range. Metals have low resistivity, in the range of $10^{-8}$ Ω m to $10^{-6}$ Ω m. Insulators like glass and rubber have $10^{22}$ to $10^{24}$ times greater resistivity. Semiconductors like Si and Ge lie roughly in the middle range of resistivity on a logarithmic scale.

  The basic difference between resistance and resistivity is that resistance depends upon the nature of the material as well as the dimensions of conductor whereas resistivity depends only upon the nature of material.

- **Conductivity (σ)**: It is the reciprocal of resistivity.

  For **ohmic conductors,** one can note that the (V-I) graphs start from the origin and are straight lines, inclined at a certain angle to the axes.

  For **non-ohmic conductors**, the (V-I) graphs may or may not start from origin and are non-linear.

  (a) Ohm's law is obeyed by many substances, but it is not a fundamental law of nature. It fails if V depends on I non-linearly.

  (b) The relation between V and I depends on the sign of V for the same absolute value of V.

  (c) The relation between V and I is non-unique.

- **Thermal Coefficient of Resistance (α):** It is the fractional change in resistance per unit change in temperature. The value of α is different for different types of materials, like
  - For metals α is positive
  - For non-ohmic conductors like insulators , semiconductor and electrolytes α is negative
  - Metal alloys like manganin, constantan and nichrome have high resistivity and 'α' is very small and positive for them.

    The dependence of **resistivity** with change in temperature is given by:

    $$\rho_2 = \rho_1(1 + \alpha \Delta T)$$

    The dependence of **resistance** with change in temperature is given by:

    $$R_2 = R_1(1 + \alpha \Delta QT)$$

- **Energy:** It is defined as the ability to do work. More is the work done, more is the energy. Electrical energy gained by a charge q, when accelerated through a potential difference V is given by. $W = q\Delta V$

- **Power:** It is defined as the rate of doing work. $P = W/t$